$S ::= x := e \mid \text{skip} \mid S; S \mid \text{if } b \text{ then } S \text{ else } S \mid \text{while } b \text{ do } S$

$e ::= n \mid x \mid e + e \mid e - e \mid e * e$

$b ::= \text{true} \mid \text{false} \mid \neg b \mid b \wedge b \mid e = e \mid e \leq e$

$$\langle x := x+1,\ \sigma_0 \rangle \to \sigma_0[x \mapsto \underline{4}]$$

$$\sigma_0 = [x \mapsto \underline{3},\ y \mapsto \underline{0},\ z \mapsto \underline{0}]$$

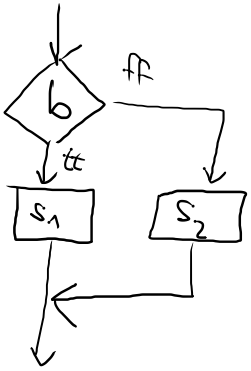
$$\sigma' = \sigma_0[x \mapsto \underline{4}]$$

$$\sigma' = [x \mapsto \underline{4},\ y \mapsto \underline{0},\ z \mapsto \underline{0}]$$

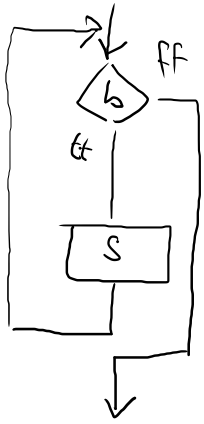
$$\frac{\langle S_1, \sigma \rangle \to \sigma' \qquad \langle S_2, \sigma' \rangle \to \sigma''}{\langle S_1; S_2, \sigma \rangle \to \sigma''}$$

$$\frac{\langle z := x, \sigma \rangle \to \sigma_1 \qquad \dfrac{\langle x := y, \sigma_1 \rangle \to \sigma_2 \quad \langle y := z, \sigma_2 \rangle \to \sigma'}{\langle x := y;\ y := z, \sigma_1 \rangle \to \sigma'}}{\langle z := x;\ x := y;\ y := z, \sigma \rangle \to \sigma'}$$

if



while



$$\frac{\langle S, s\rangle \to s'' \quad \langle \text{while } b \text{ do } S, s''\rangle \to s' \quad \mathcal{B}[\![b]\!]s = tt}{\langle \text{while } b \text{ do } S, s\rangle \to s'}$$

$$\frac{\mathcal{B}[\![b]\!]s = ff}{\langle \text{while } b \text{ do } S, s\rangle \to s}$$

while $\neg(x=1)$ do $x := x-1$ if $\begin{cases} \text{terminates if } x \geq 1 \\ \text{loops if } x < 1 \end{cases}$

while $(1 \leq x)$ do $x := x-1$ terminates always

while true do (...)

Semantic equivalence

while b do S

if b then (S; while b do S) else skip

Structural operational semantics

$$\langle S_1, \sigma \rangle \Rightarrow \sigma'$$
$$\langle S_1, \sigma \rangle \Rightarrow \langle S_1', \sigma' \rangle$$

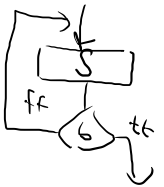
$$\frac{\langle S_1, \sigma \rangle \Rightarrow \sigma'}{\langle S_1; S_2, \sigma \rangle \Rightarrow \langle S_2, \sigma' \rangle}$$

$$\frac{\langle S_1, \sigma \rangle \Rightarrow \langle S_1', \sigma' \rangle}{\langle S_1; S_2, \sigma \rangle \Rightarrow \langle S_1'; S_2, \sigma' \rangle}$$

Example: while $(1 \leq x)$ do $(y := y * x;\ x := x - 1)$; $fac := y$;

repeat S until b

$S ; \text{while } \neg b \text{ do } S$



NATURAL SEMANTICS

$$\frac{\langle S, s\rangle \to s' \qquad \mathcal{B}[\![b]\!]s' = tt}{\langle \text{repeat } S \text{ until } b, s\rangle \to s'}$$

$$\frac{\langle S, s\rangle \to s' \quad \langle \text{repeat } S \text{ until } b, s'\rangle \to s'' \qquad \mathcal{B}[\![b]\!]s' = ff}{\langle \text{repeat } S \text{ until } b, s\rangle \to s''}$$

STRUCTURAL OPERATIONAL SEMANTICS

$\langle \text{repeat } S \text{ until } b, s\rangle \Rightarrow$

$\Rightarrow \langle S ; \text{if } b \text{ then skip else repeat } S \text{ until } b, s\rangle$

FOR - loop

for $x := e_1$ to e_2 do S

$x := e_1$;
while $(x \leq e_2)$ do $(S ;\ x := x+1)$

NATURAL SEMANTICS

$$\frac{\langle x := e_1, s\rangle \to s' \quad \langle S, s'\rangle \to s'' \quad \langle \text{for } x := e_3 \text{ to } e_2 \text{ do } S, s''\rangle \to s''' \quad \mathcal{B}[\![\neg(x = e_2)]\!]s' = tt}{\langle \text{for } x := e_1 \text{ to } e_2 \text{ do } S, s\rangle \to s'''}$$

$e_3 = \mathcal{N}^{-1}[\![\mathcal{E}[\![e_1]\!]s' \oplus 1]\!]$

$$\frac{\langle x := e_1, s\rangle \to s' \quad \mathcal{B}[\![\neg(x = e_2)]\!]s' = ff}{\langle \text{for } x := e_1 \text{ to } e_2 \text{ do } S, s\rangle \to s'}$$

STRUCTURAL OPERATIONAL SEMANTICS

$$\frac{\langle x := e_1, s\rangle \Rightarrow s'}{\langle \text{for } x := e_1 \text{ to } e_2 \text{ do } S, s\rangle \Rightarrow \langle \text{if } (x = e_2) \text{ then } (S ;\ \text{for } x := e_3 \text{ to } e_2 \text{ do } S) \text{ else skip}, s'\rangle}$$

ZX SPECTRUM BASIC

```
10 LET x = 1
20 FOR N = 1 TO 7 STEP 2
30 LET x = x * N
40 NEXT N
```

|| end-for